

**BASIC CONCEPTS AND PROPERTIES**

6

*Fluid – definition, distinction between solid and fluid - Units and dimensions - Properties of fluids - density, specific weight, specific volume, specific gravity, temperature, viscosity, compressibility, vapour pressure, capillary and surface tension - Fluid statics: concept of fluid static pressure, absolute and gauge pressures - pressure measurements by manometers and pressure gauges.*

**What is fluid mechanics?**

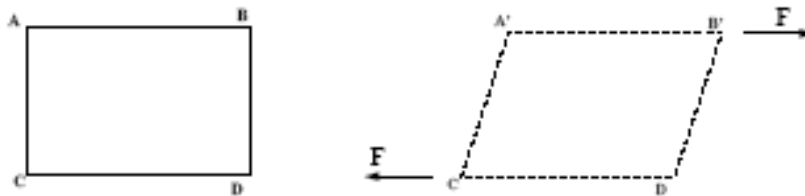
As its name suggests it is the branch of applied mechanics concerned with the statics and dynamics of fluids - both liquids and gases. The analysis of the behaviour of fluids is based on the fundamental laws of mechanics which relate continuity of mass and energy with force and momentum together with the familiar solid mechanics properties.

**Objectives of this section**

- Define the nature of a fluid.
- Show where fluid mechanics concepts are common with those of solid mechanics and indicate some fundamental areas of difference.
- Introduce viscosity and show what are Newtonian and non-Newtonian fluids
- Define the appropriate physical properties and show how these allow differentiation between solids and fluids as well as between liquids and gases.

**What makes fluid mechanics different to solid mechanics?**

- **Liquid and gas, both are known as fluid.**
- **Fluids are clearly different to solids.**
- **But we must be specific.**
- **Fluids *flow* under the action of a force, and the solids don't - but solids do deform.**
- **fluids lack the ability of solids to resist deformation.**
- **fluids change shape as long as a force acts.**



**Forces acting along edges (faces), such as  $F$ , are known as *shearing forces*.**

We can then say:

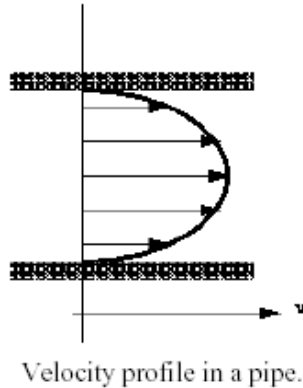
**A Fluid is a substance which deforms continuously,  
or flows, when subjected to shearing forces.**

and conversely this definition implies the very important point that:

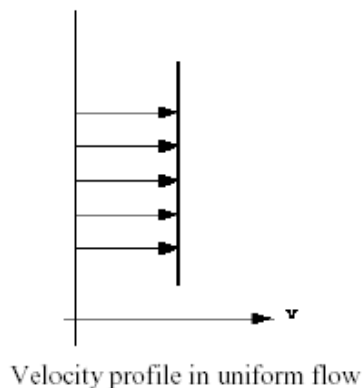
**If a fluid is at rest there are no shearing forces acting.  
All forces must be perpendicular to the planes in which they are acting.**

### Fluids in motion

Consider a fluid flowing near a wall - in a pipe for example - Fluid next to the wall will have *zero* velocity.



- The fluid “sticks” to the wall.
- Moving away from the wall velocity increases to a maximum.
- Plotting the velocity across the section gives “velocity profile”
- Change in velocity with distance is “velocity gradient” =  $du / dy$
- As fluids are usually stuck near surfaces there is usually a velocity gradient.
- Under normal conditions one fluid particle has a velocity different to its neighbour.
- Particles next to each other with different velocities exert forces on each other (due to intermolecular action ) .....
- i.e. shear forces exist in a fluid moving close to a wall.
- What if not near a wall?



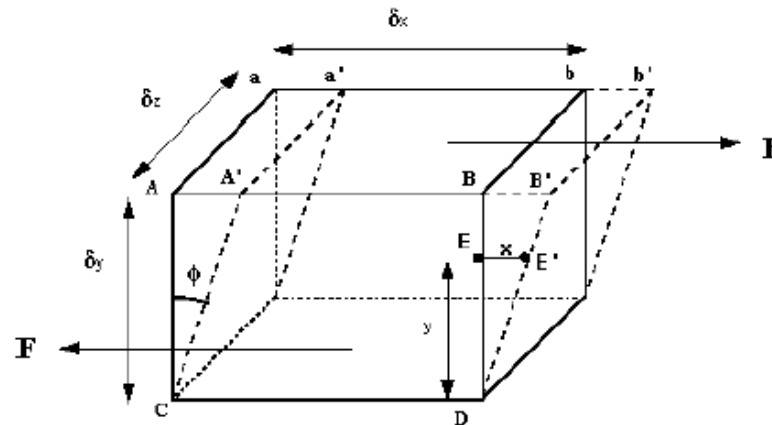
**No velocity gradient, no shear forces.**

What use is this observation?

It would be useful if we could quantify this shearing force.

We will examine the force required to deform an element.

Consider this 3-d rectangular element, under the action of the force  $F$ .



Fluid element under a shear force

The shearing force,  $F$ , acts on the area  $A = \delta z \times \delta x$

Shear stress,  $\tau$  is the force per unit area:  $\tau = F / A$

The deformation which *shear stress* causes is measured by the angle  $\phi$ , and is known as *shear strain*.

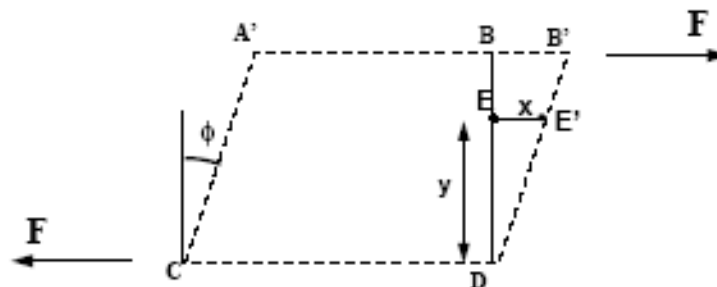
Using these definitions we can amend our definition of a fluid:

- In a fluid  $\phi$  increases for as long as  $\tau$  is applied - the fluid flows.
- In a solid shear strain,  $\phi$ , is constant for a fixed shear stress  $\tau$ .

It has been shown experimentally that the *rate of shear strain* is directly proportional to *shear stress*.

$$\tau \propto \frac{\phi}{\text{time}}$$

$$\tau = \text{cons} \tan t \times \frac{\phi}{\text{time}}$$



If a particle at point E moves to point E' in time  $t$  then: for small deformations,

$$\text{Shear strain } \phi = x / y$$

$$\text{Rate of shear strain} = \phi / t = x / (y.t) = u / y$$

(note that  $x / t = u$  is the velocity of the particle at E)

Now,

$$\tau \propto u / y$$

$u/y$  is the rate of change of velocity with distance,

or in differential form

$$\tau \propto du/dy$$

Hence,

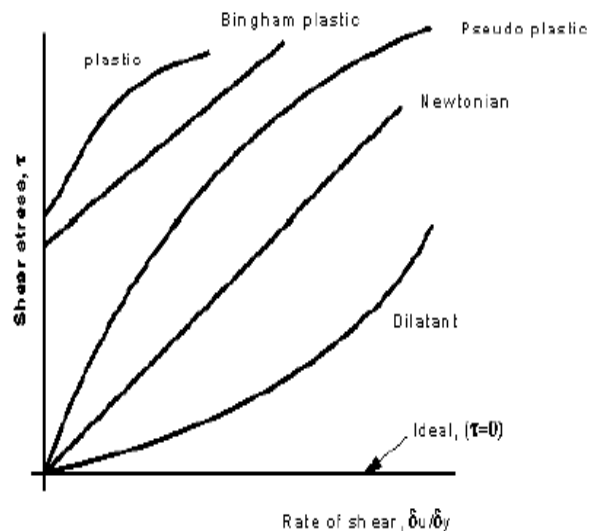
$$\tau = \text{constant} \times du/dy = \mu \times du/dy$$

Which is known as **Newton's law of viscosity**. The constant of proportionality is known as the **dynamic viscosity**,  $\mu$ .

A fluid which obeys this rule is known as a **Newtonian Fluid** (sometimes also called *real fluids*). Newtonian fluids have constant values of  $\mu$ .

### Non-Newtonian Fluids

Some fluids do not have constant  $\mu$ . They do not obey Newton's Law of viscosity. They do obey a similar relationship and can be placed into several clear categories



Shear stress vs. Rate of shear strain  $\delta u / \delta y$

The general relationship is:

$$\tau = A + B \left( \frac{\delta u}{\delta y} \right)^n$$

Where A, B and n are constants.

For Newtonian fluids  $A = 0$ ,  $B = \mu$  and  $n = 1$ .

- *Plastic*: Shear stress must reach a certain minimum before flow commences.
- *Bingham plastic*: As with the plastic above a minimum shear stress must be achieved. With this classification  $n = 1$ . An example is sewage sludge.
- *Pseudo-plastic*: No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. colloidal substances like clay, milk and cement.
- *Dilatant substances*: Viscosity increases with rate of shear e.g. quicksand.
- *Thixotropic substances*: Viscosity decreases with length of time shear force is applied e.g. thixotropic jelly paints.
- *Rheopectic substances*: Viscosity increases with length of time shear force is applied
- *Viscoelastic materials*: Similar to Newtonian but if there is a sudden large change in shear they behave like plastic

### Fluids vs. Solids

In the above we have discussed the differences between the behaviour of solids and fluids under an applied force. Summarising, we have;

- For a **solid** the strain is a function of the applied stress (providing that the elastic limit has not been reached). For a **fluid**, the rate of strain is proportional to the applied stress.
- The strain in a **solid** is independent of the time over which the force is applied and (if the elastic limit is not reached) the deformation disappears when the force is removed. A **fluid** continues to flow for as long as the force is applied and will not recover its original form when the force is removed.

### Liquids vs. Gasses

Although liquids and gasses behave in much the same way and share many similar characteristics, they also possess distinct characteristics of their own. Specifically

- A liquid is difficult to compress and often regarded as being incompressible.
- A gas is easily to compress and usually treated as such - it changes volume with pressure.
- A given mass of liquid occupies a given volume and will occupy the container it is in and form a free surface (if the container is of a larger volume).
- A gas has no fixed volume, it changes volume to expand to fill the containing vessel. It will completely fill the vessel so no free surface is formed.

### Continuum

Continuum  $\Rightarrow$  continuous distribution of matter with no empty space.

This assumption is justifiable because the number of molecules involved is vast and distance between them so small.

The properties of fluid may be adequately accounted for in their overall effect by ascribing fluid to be continuum.

Whether fluid is continuum or not can be defined by a number called Knudsen number which is defined as

$$K_n = \frac{\lambda}{L} = \frac{\text{mean free path}}{\text{characteristic dimension}}$$

If  $K_n < 0.1$ , then the fluid is continuum.

$K_n > 10$ , then the fluid consists of free molecules.

### **Real and Ideal Fluid**

Ideal Fluid  $\Rightarrow$  inviscid and incompressible. No tangential forces (shear forces) exist; only normal forces (pressure) do.

### **Properties of Fluids:**

1. **Density**  $\Rightarrow$  There are three ways of expressing density:

i) **Mass density:**

$\rho$  = mass of substance per unit volume

Units: Kilograms per cubic metre,  $kg/m^3$  (or  $kgm^{-3}$ )

Dimensions:  $ML^{-3}$

Typical values:

Water =  $1000 kgm^{-3}$ , Mercury =  $13546 kgm^{-3}$ , Air =  $1.23 kgm^{-3}$ , Paraffin Oil =  $800 kgm^{-3}$ .

(at pressure =  $1.013 \times 10^5 N m^{-2}$  and Temperature = 288.15 K.)

ii) **Specific Weight**

Specific Weight  $\gamma$ , (sometimes  $w$ , and sometimes known as *specific gravity*) is defined as the weight per unit volume or The force exerted by gravity,  $g$ , upon a unit volume of the substance.

The Relationship between  $g$  and  $\gamma$  can be determined by Newton's 2<sup>nd</sup> Law, since

$$\begin{aligned} \text{weight per unit volume} &= \text{mass per unit volume} \times g \\ \gamma &= \rho g \end{aligned}$$

Units: Newton's per cubic metre,  $N/m^3$  (or  $N m^{-3}$ )

Dimensions:  $ML^{-2} T^{-2}$ .

Typical values:

Water =  $9814 Nm^{-3}$ , Mercury =  $132943 Nm^{-3}$ , Air =  $12.07 Nm^{-3}$ , Paraffin Oil =  $7851 Nm^{-3}$

iii) **Relative Density or Specific Gravity**

Relative Density,  $\sigma$  or  $s$ , is defined as the ratio of mass density of a substance to some standard mass density.

For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4<sup>0</sup>C) at atmospheric pressure.

$$\sigma = \frac{\rho_{\text{substance}}}{\rho_{\text{water at 4 deg. C}}}$$

Units: None, since a ratio is a pure number.

Dimensions: 1.

Typical values: Water = 1, Mercury = 13.5, Paraffin Oil = 0.8.

### Specific Volume

The term is used for the reciprocal of density and denoted by  $v$ .

Unit: m<sup>3</sup>/kg

Dimension: L<sup>3</sup>M<sup>-1</sup>.

### Viscosity

Viscosity,  $\mu$ , is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to shear deformation. Different fluids deform at different rates under the same shear stress.

Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water.

All fluids are viscous, “Newtonian Fluids” obey the linear relationship given by Newton’s law of viscosity,  $\tau = \mu \, du/dy$ , which we saw earlier.

Units  $N \, m^{-2}$ ;  $kg \, m^{-1} \, s^{-2}$

Dimensions  $ML^{-1} T^{-2}$ .

$du/dy$  is the velocity gradient or rate of shear strain, and has

Units:  $radians \, s^{-1}$ ,

Dimensions  $t^{-1}$

$\mu$  is the “coefficient of dynamic viscosity” - see below.

### Coefficient of Dynamic Viscosity

The Coefficient of Dynamic Viscosity,  $\mu$ , is defined as the shear force, per unit area, (or shear stress  $\tau$ ), required to drag one layer of fluid with unit velocity past another layer a unit distance away.

$$\mu = \frac{\tau}{\frac{\delta u}{\delta y}} = \frac{\frac{\text{Force}}{\text{Area}}}{\frac{\text{Velocity}}{\text{Distance}}} = \frac{\text{Force} \times \text{Time}}{\text{Area}} = \frac{\text{Mass}}{\text{Length} \times \text{Area}}$$

Units : Newton seconds per square meter,  $\text{N s m}^{-2}$  or Kilograms per meter per second,  $\text{kg m}^{-1} \text{s}^{-1}$ .

Typical values:

Water =  $1.14 \times 10^{-3} \text{ kg m}^{-1} \text{s}^{-1}$ , Air =  $1.78 \times 10^{-5} \text{ kg m}^{-1} \text{s}^{-1}$ , Mercury =  $1.552 \text{ kg m}^{-1} \text{s}^{-1}$ , Paraffin oil =  $1.9 \text{ kg m}^{-1} \text{s}^{-1}$ .

In CGS, the unit of viscosity is Poise (P).

$1 \text{ N s/m}^2 = 10 \text{ P}$ .

### Causes of Viscosity in Fluids

#### *a) Viscosity in Gasses*

The molecules of gasses are only weakly kept in position by molecular cohesion (as they are so far apart). As adjacent layers move by each other there is a continuous exchange of molecules. Molecules of a slower layer move to faster layers causing a drag, while molecules moving the other way exert an acceleration force. Mathematical considerations of this momentum exchange can lead to Newton law of viscosity.

If temperature of a gas increases the momentum exchange between layers will increase thus increasing viscosity.

Viscosity will also change with pressure - but under normal conditions this change is negligible in gasses.

#### *b) Viscosity in Liquids*

There is some molecular interchange between adjacent layers in liquids - but as the molecules are so much closer than in gasses the cohesive forces hold the molecules in place much more rigidly. This cohesion plays an important roll in the viscosity of liquids.

Increasing the temperature of a fluid reduces the cohesive forces and increases the molecular interchange. Reducing cohesive forces reduces shear stress, while increasing molecular interchange increases shear stress. Because of this complex interrelation the effect of temperature on viscosity has something of the form:

$$\mu_T = \mu_0 / (1 + AT + BT^2)$$

where  $\mu_T$  is the viscosity at temperature  $T^\circ\text{C}$ , and  $\mu_0$  is the viscosity at temperature  $0^\circ\text{C}$ . A and B are constants for a particular fluid.

High pressure can also change the viscosity of a liquid. As pressure increases the relative movement of molecules requires more energy hence viscosity increases.

### Kinematic Viscosity

Kinematic Viscosity,  $\nu$ , is defined as the ratio of dynamic viscosity to mass density.



$$v = \frac{\mu}{\rho}$$

Units:  $\text{m}^2\text{s}^{-1}$

(Although note that  $v$  is often expressed in Stokes, S, where  $10^4 \text{ S} = 1 \text{ m}^2\text{s}^{-1}$ .)

Dimension:  $\text{L}^2\text{T}^{-1}$ .

Typical values:

Water =  $1.14 \times 10^{-6} \text{ m}^2\text{s}^{-1}$ , Air =  $1.46 \times 10^{-5} \text{ m}^2\text{s}^{-1}$ , Mercury =  $1.145 \times 10^{-4} \text{ m}^2\text{s}^{-1}$ ,  
Paraffin oil =  $2.375 \times 10^{-3} \text{ m}^2\text{s}^{-1}$ .

In CGS,  $\text{cm}^2/\text{s}$  or Stokes (S) is used as unit for kinematic viscosity.

1 centistokes (cS) =  $10^{-2} \text{ S} = 1 \text{ mm}^2/\text{s}$ .

### Vapour Pressure

All liquids tend to evaporate. This is because there is at the free surface a continual movement of molecules moving out of the liquid. Some of these molecules return to the liquid. If the space above the surface is enclosed, the number of molecules in the space will increase until the rate at which the molecules escape from the liquid is balanced by the rate at which they return to it.

Just above the liquid surface, the molecules returning to the liquid create a pressure known as the ***partial pressure of vapour***. Molecules leaving the liquid give rise to the ***vapour pressure***. When the vapour pressure equals to the partial pressure of vapour above the surface, the rates at which molecules leave and enter the liquid are the same. The gas above the surface is said to be saturated with the vapour and the corresponding pressure is known as ***saturation pressure***.

### The Perfect gas: Equation of State

No actual gas is perfect. From experimental observations it has been established that the following relation can express the  $p - v - T$  behaviour of gases at the low pressure

$$pV = mRT$$

i.e.

$$p = \rho RT$$

where,

$p$  = absolute pressure,  $V$  = volume occupied by mass  $m$ ,  $T$  = absolute temperature,  
 $\rho$  = density and  $R$  = gas constant.

Any equation which relates  $p$ ,  $\rho$  and  $T$  is known as ***equation of state***.

Any gas for which  $p/\rho T = \text{constant}$  is said to be ***thermally perfect***.

A gas for which specific heat at constant volume  $C_v$  is constant is said to be ***calorically perfect***.

Compressibility

All matter is to some extent is compressible. That is, a change in compressive stress applied to a body always produces some change in volume. The degree of compressibility of a substance is characterised by **bulk modulus of elasticity**, **K**, which is defined by

$$K = - \frac{\delta p}{\delta V / V}$$

Here,  $\delta p$  represents a small increase of pressure applied to material and  $\delta V$  the corresponding small increase in volume. Note that  $\delta V$  is always negative and the minus sign is included so that value of  $K$  is always positive.

As  $\delta p \rightarrow 0$ ,

$$K = - \frac{V \partial p}{\partial V} = \frac{\partial p}{\frac{\partial \rho}{\rho}} = \frac{\rho \partial p}{\partial \rho}$$

For liquid the bulk modulus is very high so that the change in density with increase in pressure is very small. So the density of liquid can be taken as constant i.e., liquids are incompressible.

Surface Tension

All molecules are in constant motion. A molecule within the body of the liquid is attracted equally in all directions by other molecules surrounding it. At the surface between the liquid and air, or interface between one substance and another, the upward and downward attractions are unbalanced, the surface molecules being pulled inward towards the bulk liquid. The liquid surface behaves as if it were an elastic membrane under tension.

Surface tension is denoted as  $\sigma$  and expressed as N/m.

For a spherical liquid droplet of radius  $r$ ,

$$\text{Force due to internal pressure, } p = p \times \pi r^2.$$

$$\text{Force due to surface tension around the perimeter} = 2\pi r \times \sigma.$$

For equilibrium

$$p \pi r^2 = 2\pi r \sigma \quad \text{or} \quad p = 2\sigma/r.$$

For a hollow bubble like a soap bubble, air has two surfaces in contact with, one inside and other outside. Thus two surfaces are subjected to surface tension. So

$$p \pi r^2 = 2 \times (2\pi r \sigma) \quad \text{or} \quad p = \frac{4\sigma}{r}$$

For a liquid jet of radius  $r$  and length  $L$ ,

$$\text{Force due to internal pressure, } p = p \times L \times 2r.$$

$$\text{Force due to surface tension around the perimeter} = 2L \times \sigma.$$

Equating, we get

$$p \times L \times 2r = 2L \times \sigma \quad \text{or} \quad p = \sigma/r$$

### Capillary

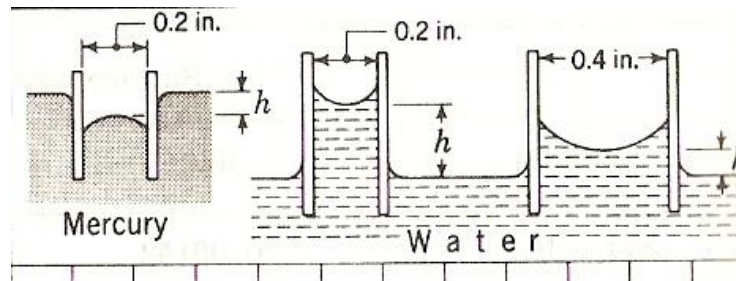
If a fine tube, open at both ends, is lowered vertically into a liquid which wets the tube, the level of the liquid will rise in the tube. If the liquid does not wet the tube, the level of liquid will be depressed below the level of the free surface outside. If  $\theta$  is the angle of contact between liquid and solid and  $d$  is the tube diameter

Upward pull due to surface tension = component of surface tension acting upwards x perimeter of tube  
 $= \sigma \cos\theta \times \pi d$ .

The weight of liquid column of height  $h$  in the tube  $= \frac{\pi}{4} d^2 \times h \times \rho \times g$

Therefore,  $\frac{\pi}{4} d^2 h \rho g = \sigma \cos \theta \pi d$

Capillary rise,  $h = \frac{4 \sigma \cos \theta}{\rho g d}$



### Phenomena of capillary effect

This is due to cohesive forces, i.e. forces between like molecules like water and water, and adhesive forces, i.e. the forces between unlike molecules such as water and glass. The liquid molecules at the solid-liquid interface are subjected to both cohesive forces by other liquid molecules and adhesive forces by the molecules of solid. The relative magnitude of these forces determine whether a liquid wets a solid surface (i.e. whether  $\theta < 90^\circ$ ) or not (i.e. whether  $\theta > 90^\circ$ ). Water molecules are more strongly attracted to the glass molecules than they are to other water molecules. So water tends to rise along the glass surface. For Mercury, opposite is true i.e. the cohesive forces are more than the adhesive forces. Hence, we get depression for Mercury in glass.

In atmospheric air,

$\theta \approx 0^\circ$  for water with glass,

$\theta \approx 130^\circ$  for mercury with glass

$\theta \approx 26^\circ$  for kerosene with glass

**Surface tension of some fluids in air at 1 atm. and 20°C.**

Fluid	Surface tension, $\sigma$ N/m
Water	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
Mercury	0.440
Ethyl Alcohol	0.023
Soap solution	0.025
Kerosene	0.028

Surface tension decreases with temperature. It changes considerably with impurities. Soap and detergent lowers the surface tension of water.

## Forces in Static Fluids

This section will study the forces acting on or generated by fluids at rest.

### Objectives

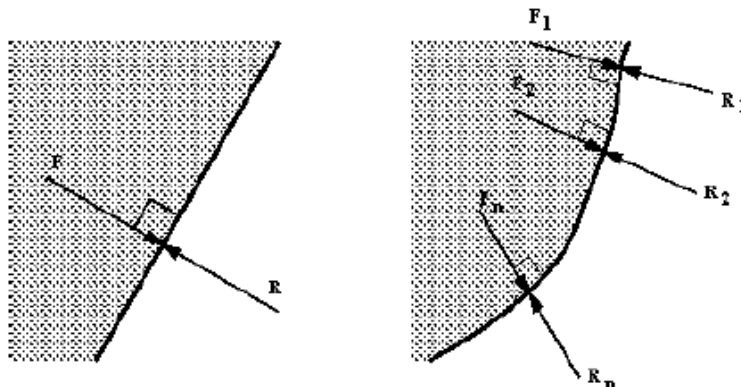
- Introduce the concept of pressure;
- Prove it has a unique value at any particular elevation;
- Show how it varies with depth according to the hydrostatic equation and
- Show how pressure can be expressed in terms of *head* of fluid.

This understanding of pressure will then be used to demonstrate methods of pressure measurement that will be useful later with fluid in motion.

### Fluids statics

The general rules of statics (as applied in solid mechanics) apply to fluids at rest. From earlier we know that:

- a static fluid can have **no shearing force** acting on it, and that
- any force between the fluid and the boundary must be acting at right angles to the boundary.



Pressure force normal to the boundary

Note that this statement is also true for curved surfaces, in this case the force acting at any point is normal to the surface at that point. The statement is also true for any imaginary plane in a static fluid. We use this fact in our analysis by considering elements of fluid bounded by imaginary planes.

We also know that:

- For an element of fluid at rest, the element will be in equilibrium - the sum of the components of forces in any direction will be zero.
- The sum of the moments of forces on the element about any point must also be zero.

It is common to test equilibrium by resolving forces along three mutually perpendicular axes and also by taking moments in three mutually perpendicular planes and to equate these to zero.

### Pressure

As mentioned above a fluid will exert a normal force on any boundary it is in contact with. Since these boundaries may be large and the force may differ from place to place it is convenient to work in terms of pressure,  $p$ , which is the force per unit area.

If the force exerted on each unit area of a boundary is the same, the pressure is said to be *uniform*.

$$\text{pressure} = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

$$\text{or } p = \frac{F}{A}$$

Units:  $\text{N/m}^2$ ,  $\text{kg/ms}^2$

The same unit is known as Pascal, Pa i.e.  $1 \text{ Pa} = 1 \text{ N/m}^2$ .

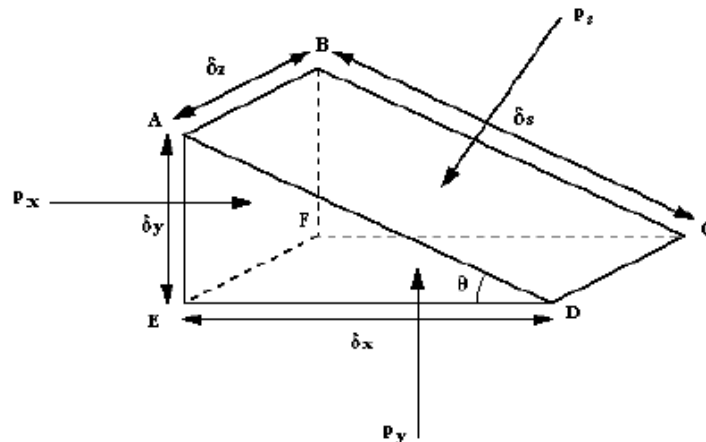
(Alternative unit in CGS is bar.  $1 \text{ bar} = 10^5 \text{ N/m}^2$ .)

Dimension:  $\text{M L}^{-1}\text{T}^{-2}$ .

### Pascal's Law for Pressure at a Point

(Proof that pressure acts equally in all directions.)

By considering a small element of fluid in the form of a triangular prism which contains a point P, we can establish a relationship between the three pressures  $p_x$  in the x direction,  $p_y$  in the y direction and  $p_s$  in the direction normal to the sloping face.



Triangular prismatic element of fluid

The fluid is at rest, so we know there are no shearing forces, and we know that all forces are acting at right angles to the surfaces i.e.  $p_s$  acts perpendicular to surface ABCD,  $p_x$  acts perpendicular to surface ABEF and  $p_y$  acts perpendicular to surface FECD.

And, as the fluid is at rest, in equilibrium, the sum of the forces in any direction is zero.  
Summing forces in the x-direction:

Force due to  $p_x$ ,

$$\begin{aligned} F_x &= p_x \times \text{Area ABFE} - \text{component } p_s \text{ in x-direction} \times \text{Area ABCD} \\ &= p_x \delta y \delta z - p_s \delta s \delta z \sin\theta = p_x \delta y \delta z - p_s \delta s \delta z \delta y / \delta s \\ &= p_x \delta y \delta z - p_s \delta y \delta z \end{aligned}$$

To be at rest (in equilibrium)

$$\begin{aligned} F_x &= 0 \\ \text{or } p_x \delta y \delta z - p_s \delta y \delta z &= 0 \\ \text{or } p_x &= p_s \end{aligned}$$

Similarly, summing forces in the y-direction:

Force due to  $p_y$ ,

$$\begin{aligned} F_y &= p_y \times \text{Area EFCD} - \text{component } p_s \text{ in y-direction} \times \text{Area ABCD} - \text{weight of fluid} \\ &= p_y \delta x \delta z - p_s \delta s \delta z \cos\theta - \rho g \frac{1}{2} \delta x \delta y \delta z \\ &= p_y \delta x \delta z - p_s \delta s \delta z \delta x / \delta s - \rho g \frac{1}{2} \delta x \delta y \delta z \\ &= p_y \delta x \delta z - p_s \delta x \delta z - \rho g \frac{1}{2} \delta x \delta y \delta z \end{aligned}$$

To be at rest (in equilibrium)

$$\begin{aligned} F_y &= 0 \\ \text{or } p_y \delta x \delta z - p_s \delta x \delta z - \rho g \frac{1}{2} \delta x \delta y \delta z &= 0 \end{aligned}$$

The element is small i.e.  $\delta x$ ,  $\delta y$  and  $\delta z$  are small, and considered negligible, hence

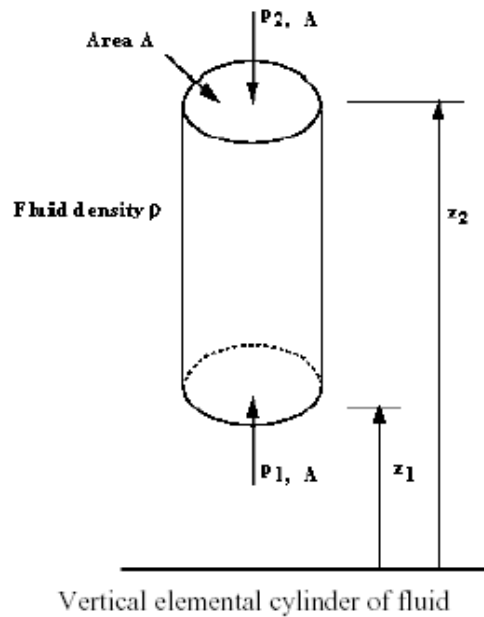
$$p_y = p_s$$

thus

$$p_x = p_y = p_s$$

Hence,

Pressure at any point is the same in all directions.  
This is known as **Pascal's Law** and applies to fluids at rest.

**Variation of Pressure Vertically in a Fluid under Gravity**

In the above figure we can see an element of fluid which is a vertical column of constant cross sectional area,  $A$ , surrounded by the same fluid of mass density  $\rho$ . The pressure at the bottom of the cylinder is  $p_1$  at level  $z_1$ , and at the top is  $p_2$  at level  $z_2$ . The fluid is at rest and in equilibrium so all the forces in the vertical direction sum to zero. i.e. we have

$$\begin{aligned}
 \text{Force due to } p_1 \text{ on } A \text{ (upward)} &= p_1 A \\
 \text{Force due to } p_2 \text{ on } A \text{ (downward)} &= p_2 A \\
 \text{Force due to weight of element (downward)} &= mg \\
 &= \text{mass density} \times \text{volume} \\
 &= \rho g A (z_2 - z_1)
 \end{aligned}$$

Taking upward as positive, in equilibrium we have

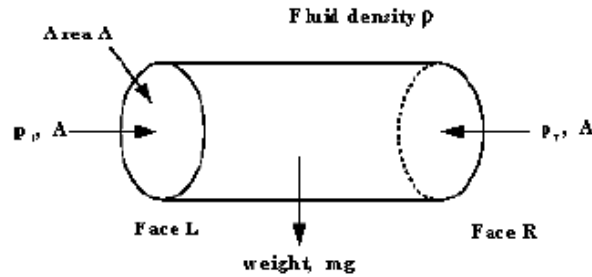
$$\begin{aligned}
 p_1 A - p_2 A &= \rho g A (z_2 - z_1) \\
 p_1 - p_2 &= \rho g (z_2 - z_1)
 \end{aligned}$$

**In a fluid pressure decreases linearly with increase in height**

**Equality of Pressure at the same level in a Static Fluid**

Consider the horizontal cylindrical element of fluid in the figure below, with cross-sectional area  $A$ , in a fluid of density  $\rho$ , pressure  $p_1$  at the left hand end and pressure  $p_2$  at the right hand end.





Horizontal elemental cylinder of fluid

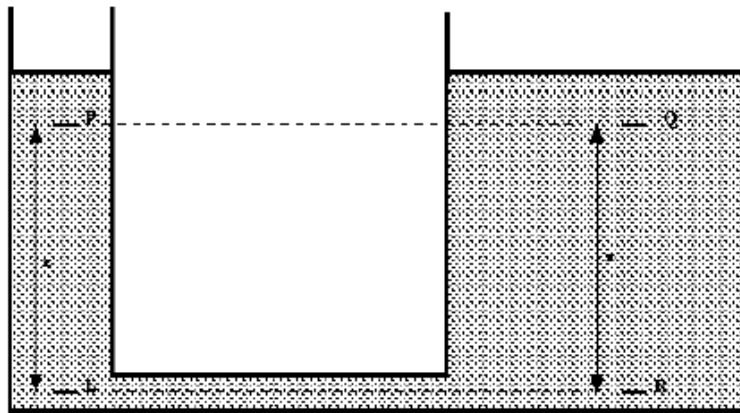
The fluid is at equilibrium so the sum of the forces acting in the x direction is zero.

$$p_l A = p_r A$$

$$p_l = p_r$$

Pressure in the horizontal direction is constant. This true for any *continuous* fluid

It is still true for two connected tanks which appear not to have any direct connection, for example consider the tank in the figure below.



Two tanks of different cross-section connected by a pipe

We have shown above that  $p_l = p_r$  and from the equation for a vertical pressure change we have

$$p_l = p_p + \rho g z$$

and

$$p_r = p_q + \rho g z$$

so

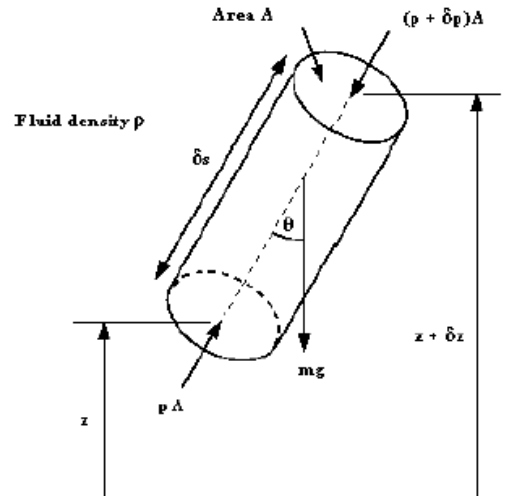
$$p_p + \rho g z = p_q + \rho g z$$

$$p_p = p_q$$

This shows that the pressures at two equal levels P and Q are same.

### **General Equation for Variation of Pressure in a Static Fluid**

Here we show how the above observations for vertical and horizontal elements of fluids can be generalised for an element of any orientation.



A cylindrical element of fluid at an arbitrary orientation.

Consider the cylindrical element of fluid in the figure above, inclined at an angle  $\theta$  to the vertical, length  $\delta s$ , cross-sectional area  $A$  in a static fluid of mass density  $\rho$ . The pressure at the end with height  $z$  is  $p$  and at the end of height  $z + \delta z$  is  $p + \delta p$ .

The forces acting on the element are

$$\begin{array}{ll}
 pA & \text{acting at right - angles to the end of the face at } z \\
 (p + \delta p)A & \text{acting at right - angles to the end of the face at } z + \delta z \\
 mg & = \rho A \delta s g
 \end{array}$$

There are also forces from the surrounding fluid acting normal to these sides of the element. For equilibrium of the element the resultant of forces in any direction is zero.

Resolving the forces in the direction along the central axis gives

$$\begin{aligned}
 pA - (p + \delta p)A - \rho g A \delta s \cos \theta &= 0 \\
 \delta p &= -\rho g \delta s \cos \theta
 \end{aligned}$$

or, in differential form

$$\frac{dp}{ds} = -\rho g \cos \theta$$

If  $\theta = 90^\circ$  then  $s$  is in the  $x$  or  $y$  directions, (i.e. horizontal), so

$$\left( \frac{dp}{ds} \right)_{\theta=90^\circ} = \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

Confirming that pressure on any horizontal plane is zero.

If  $\theta = 0^\circ$  then  $s$  is in the  $z$  direction (vertical) so

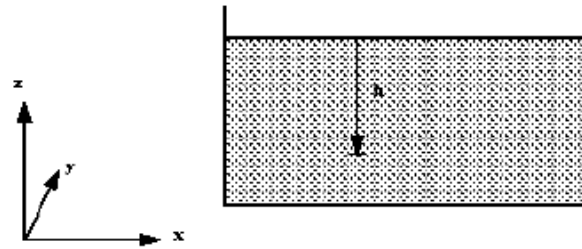
$$\left( \frac{dp}{ds} \right)_{\theta=0^\circ} = \frac{dp}{dz} = -\rho g$$

**Pressure and Head**

In a static fluid of constant density we have the relationship  $dp/dz = -\rho g$ , as shown above. This can be integrated to give

$$p = \rho g z + \text{constant}$$

In a liquid with a free surface the pressure at any depth  $z$  measured from the free surface so that  $z = -h$  (see the figure below)



Fluid head measurement in a tank.

This gives the pressure

$$p = \rho g h + \text{constant}$$

At the surface of fluids we are normally concerned with, the pressure is the atmospheric pressure,  $p_{\text{atm}}$ . So

$$p = \rho g h + p_{\text{atm}}$$

As we live constantly under the pressure of the atmosphere, and everything else exists under this pressure, it is convenient (and often done) to take atmospheric pressure as the datum. So we quote pressure as above or below atmospheric.

Pressure quoted in this way is known as gauge pressure i.e.

**Gauge pressure** is

$$p_{\text{gage}} = \rho g h$$

The lower limit of any pressure is zero - that is the pressure in a perfect vacuum. Pressure measured above this datum is known as absolute pressure i.e.

**Absolute pressure** is

$$p_{\text{abs}} = \rho g h + p_{\text{atm}}$$

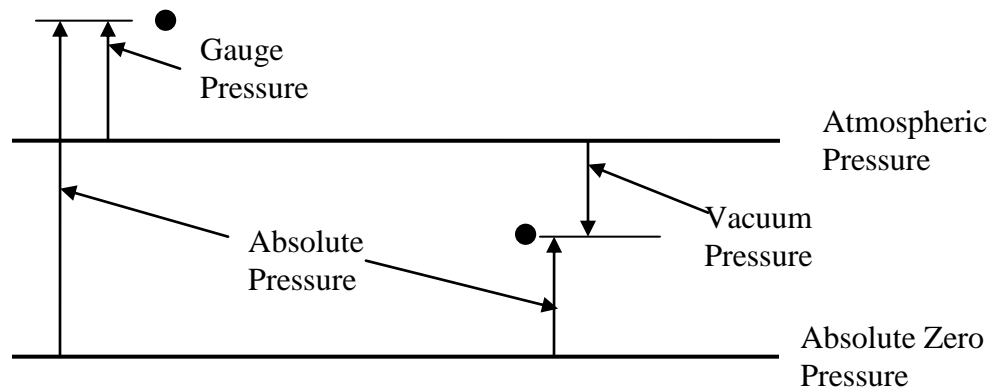
Absolute pressure = Gauge pressure + Atmospheric pressure

As  $g$  is (approximately) constant, the gauge pressure can be given by stating the vertical height of any fluid of density  $\rho$  which is equal to this pressure.

$$p = \rho g h$$

This vertical height is known as **head** of fluid.

**Note:** If pressure is quoted in *head*, the density of the fluid *must* also be given.



### Pressure Measurement by Manometer

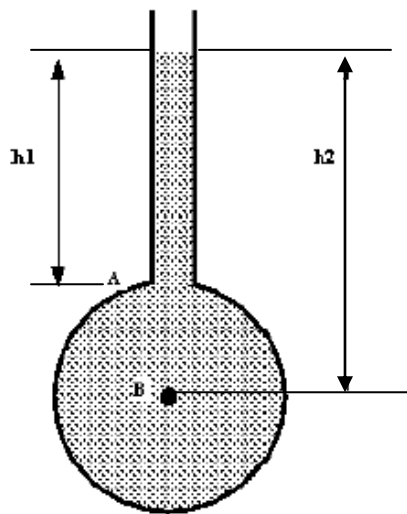
The relationship between pressure and head is used to measure pressure with a manometer (also known as a liquid gauge).

#### **Objective:**

- To demonstrate the analysis and use of various types of manometers for pressure measurement.

### The Piezometer Tube Manometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured. An example can be seen in the figure below. This simple device is known as a *Piezometer tube*. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is **gauge pressure**.



A simple piezometer tube manometer

Pressure at A = Pressure due to column of liquid of height  $h_1$ ,

$$p_A = \rho g h_1$$

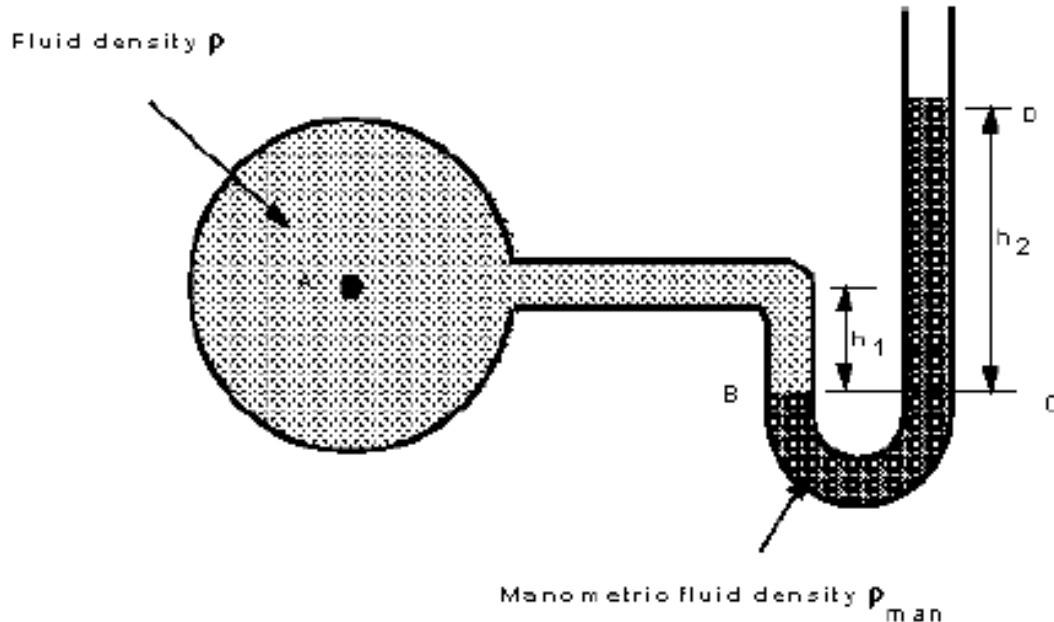
Similarly,

$$\text{Pressure at B} = p_B = \rho g h_2$$

This method can only be used for liquids (i.e. **not** for gases) and only when the liquid height is convenient to measure. It must not be too small or too large and pressure changes must be detectable.

### The “U”-Tube Manometer

Using a “U”-Tube enables the pressure of both liquids and gases to be measured with the same instrument. The “U” is connected as in the figure below and filled with a fluid called the *manometric fluid*. The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



Pressure in a continuous static fluid is the same at any horizontal level so,

$$\text{pressure at B} = \text{pressure at C}$$

$$p_B = p_C$$

For left limb,

$$\begin{aligned} p_B &= \text{Pressure } p_A \text{ at A} + \text{pressure due to depth } h_1 \text{ of fluid} \\ &= p_A + \rho g h_1 \end{aligned}$$

For the right limb,

$$\begin{aligned} p_C &= \text{Pressure } p_D \text{ at D} + \text{pressure due to depth } h_2 \text{ of fluid} \\ &= 0 + \rho_{\text{man}} g h_2 \end{aligned}$$

Since,  $p_B = p_C$

$$p_A + \rho g h_1 = \rho_{\text{man}} g h_2$$

$$p_A = \rho_{\text{man}}gh_2 - \rho gh_1$$

### What if the fluid is a gas?

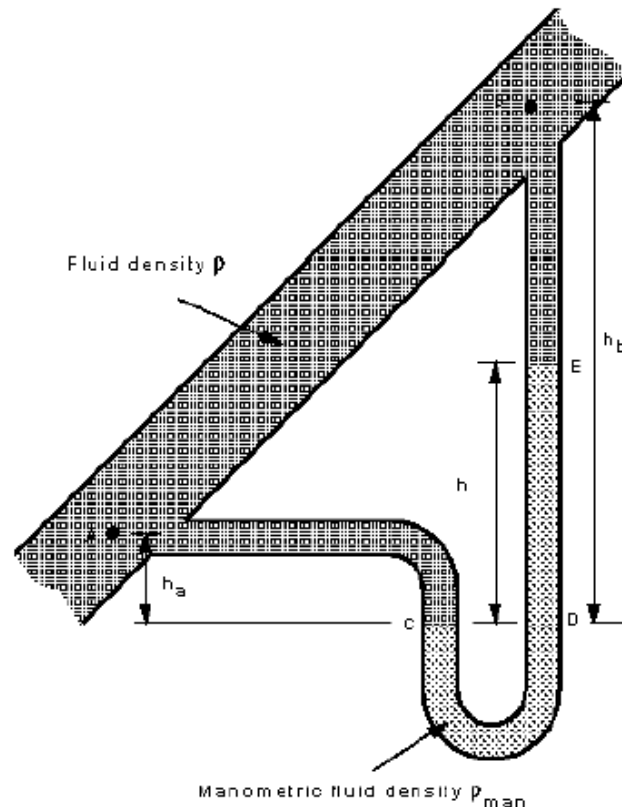
Nothing changes. The manometer work exactly the same.

BUT: As the manometric fluid is liquid (usually mercury, oil or water) and Liquid density is much greater than gas,  $\rho_{\text{man}} \gg \rho$ ;

$\rho gh_1$  can be neglected, and the gauge pressure given by  $p_A = \rho_{\text{man}}gh_2$

### Measurement of Pressure Difference using a “U”-Tube Manometer.

If the “U”-tube manometer is connected to a pressurised vessel at two points the *pressure difference* between these two points can be measured.



Pressure at C = Pressure at D

$$p_C = p_D$$

$$p_C = p_A + \rho gh_a$$

$$p_D = p_B + \rho g(h_b - h) + \rho_{\text{man}}gh$$

So,

$$p_A + \rho gh_a = p_B + \rho g(h_b - h) + \rho_{\text{man}}gh$$

or

$$p_A - p_B = \rho g(h_b - h_a) + (\rho_{\text{man}} - \rho)gh$$

**Inverted U-tube manometer**

The top of the U-tube is filled with a fluid (frequently air) which is less dense than fluid in the pipe.

Pressures at level xx will be same in both limbs.

For left limb:

$$p_{xx} = p_A - \rho g a - \rho_{man} g h$$

For the right limb:

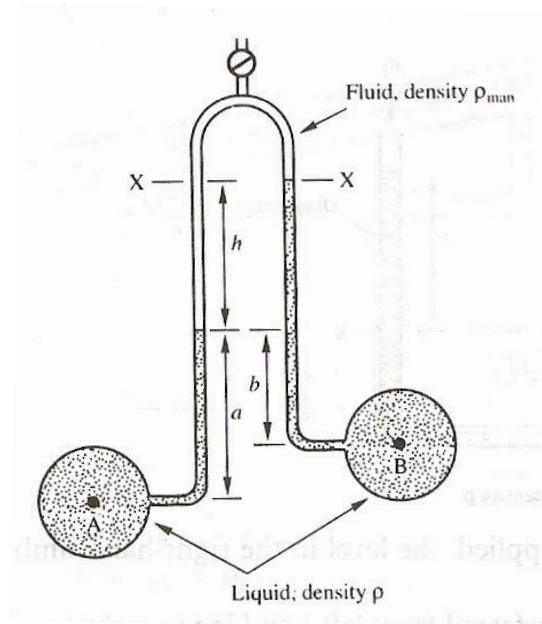
$$p_{xx} = p_B - \rho g (b+h)$$

Thus

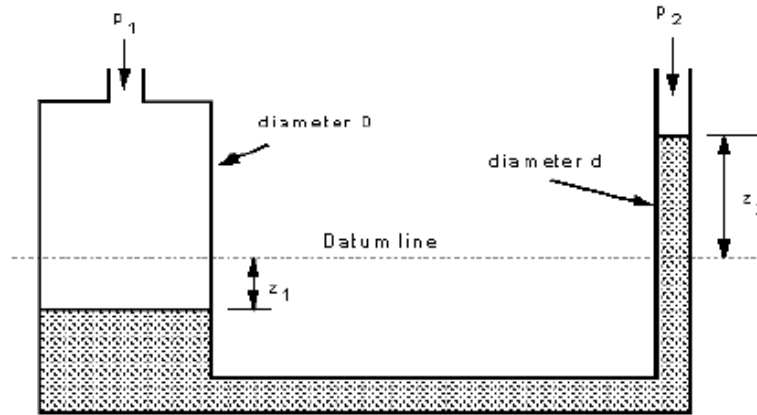
$$p_B - p_A = \rho g (b-a) + g h (\rho - \rho_{man})$$

If A and B are in the same level, then

$$p_B - p_A = (\rho - \rho_{man}) g h$$

**Advances to the “U” tube manometer.**

The “U”-tube manometer has the disadvantage that the change in height of the liquid in both sides must be read. This can be avoided by making the diameter of one side very large compared to the other. In this case the side with the large area moves very little when the small area side move considerably more.



Assume the manometer is arranged as above to measure the pressure difference of a gas of (negligible density) and that pressure difference is  $p_1 - p_2$ . If the datum line indicates the level of the manometric fluid when the pressure difference is zero and the height differences when pressure is applied is as shown, the volume of liquid transferred from the left side to the right  $= z_2 \times (\pi d^2 / 4)$

And the fall in level of the left side is

$$\begin{aligned} z_1 &= \frac{\text{Volume moved}}{\text{Area of left side}} \\ &= \frac{z_2 (\pi d^2 / 4)}{\pi D^2 / 4} \\ &= z_2 \left( \frac{d}{D} \right)^2 \end{aligned}$$

We know from the theory of the “U” tube manometer that the height different in the two columns gives the pressure difference so

$$\begin{aligned} p_1 - p_2 &= \rho g \left[ z_2 + z_2 \left( \frac{d}{D} \right)^2 \right] \\ &= \rho g z_2 \left[ 1 + \left( \frac{d}{D} \right)^2 \right] \end{aligned}$$

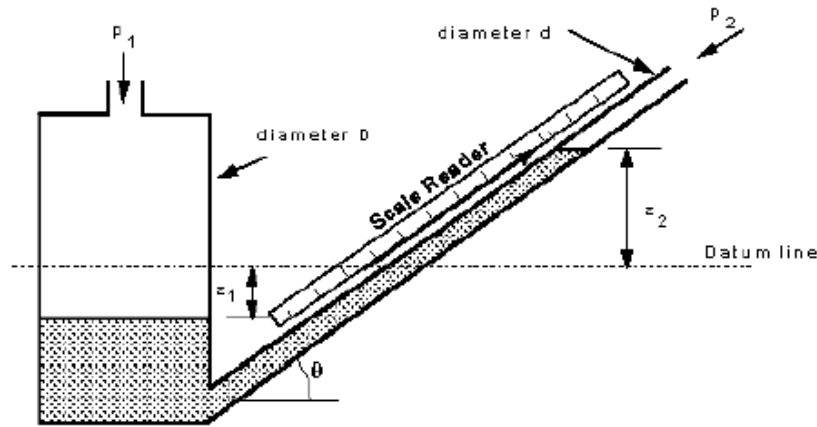
If  $D$  is very large,

$$p_1 - p_2 = \rho g z_2$$

So only one reading need be taken to measure the pressure difference.

If the pressure to be measured is very small then tilting the arm provides a convenient way of obtaining a larger (more easily read) movement of the manometer. The above arrangement with a tilted arm is shown in the figure below.





Tilted manometer.

The pressure difference is still given by the height change of the manometric fluid but by placing the scale along the line of the tilted arm and taking this reading large movements will be observed. The pressure difference is then given by

$$p_1 - p_2 = \rho g z_2 = \rho g x \sin \theta$$

The sensitivity to pressure change can be increased further by a greater inclination of the manometer arm, alternatively the density of the manometric fluid may be changed.

### Choice Of Manometer

Care must be taken when attaching the manometer to vessel, no burrs must be present around this joint.

Burrs would alter the flow causing local pressure variations to affect the measurement.

Some disadvantages of manometers:

- Slow response - only really useful for very slowly varying pressures - no use at all for fluctuating pressures;
- For the "U" tube manometer two measurements must be taken simultaneously to get the h value. This may be avoided by using a tube with a much larger cross-sectional area on one side of the manometer than the other;
- It is often difficult to measure small variations in pressure - a different manometric fluid may be required - alternatively a sloping manometer may be employed; It cannot be used for very large pressures unless several manometers are connected in series;
- For very accurate work the temperature and relationship between temperature and  $\rho$  must be known;

Some advantages of manometers:

- They are very simple.
- No calibration is required - the pressure can be calculated from first principles

Problems:

1. Explain why the viscosity of a liquid decreases while that of a gas increases with a temperature rise.

The following is a table of measurement for a fluid at constant temperature.

Determine the dynamic viscosity of the fluid.

$du/dy \text{ (s}^{-1}\text{)}$	0.0	0.2	0.4	0.6	0.8
$\tau \text{ (N m}^{-2}\text{)}$	0.0	1.0	1.9	3.1	4.0

2. The density of an oil is  $850 \text{ kg/m}^3$ . Find its relative density and Kinematic viscosity if the dynamic viscosity is  $5 \times 10^{-3} \text{ kg/ms}$ .

3. The velocity distribution of a viscous liquid (dynamic viscosity  $= 0.9 \text{ Ns/m}^2$ ) flowing over a fixed plate is given by  $u = 0.68y - y^2$  ( $u$  is velocity in m/s and  $y$  is the distance from the plate in m). What are the shear stresses at the plate surface and at  $y=0.34\text{m}$ ?

4.  $5.6\text{m}^3$  of oil weighs  $46\,800 \text{ N}$ . Find its mass density and relative density.

5. From table of fluid properties the viscosity of water is given as  $0.01008$  poises. What is this value in  $\text{Ns/m}^2$  and  $\text{Pa s}$  units?

6. In a fluid the velocity measured at a distance of  $75\text{mm}$  from the boundary is  $1.125\text{m/s}$ . The fluid has absolute viscosity  $0.048 \text{ Pa s}$  and relative density  $0.913$ . What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution.

7. What will be the (a) the gauge pressure and (b) the absolute pressure of water at depth  $12\text{m}$  below the surface?  $\rho_{\text{water}} = 1000 \text{ kg/m}^3$ , and  $p_{\text{atm}} = 101\text{kN/m}^2$ .  
[ $117.72 \text{ kN/m}^2$ ,  $218.72 \text{ kN/m}^2$ ]

8. At what depth below the surface of oil, relative density  $0.8$ , will produce a pressure of  $120 \text{ kN/m}^2$ ? What depth of water is this equivalent to?  
[ $15.3\text{m}$ ,  $12.2\text{m}$ ]

9. What would the pressure in  $\text{kN/m}^2$  be if the equivalent head is measured as  $400\text{mm}$  of (a) mercury  $\gamma = 13.6$  (b) water (c) oil specific weight  $7.9 \text{ kN/m}^3$  (d) a liquid of density  $520 \text{ kg/m}^3$ ?  
[ $53.4 \text{ kN/m}^2$ ,  $3.92 \text{ kN/m}^2$ ,  $3.16 \text{ kN/m}^2$ ,  $2.04 \text{ kN/m}^2$ ]

10. A manometer connected to a pipe indicates a negative gauge pressure of  $50\text{mm}$  of mercury. What is the absolute pressure in the pipe in Newtons per square metre if the atmospheric pressure is  $1 \text{ bar}$ ?  
[ $93.3 \text{ kN/m}^2$ ]

11. What height would a water barometer need to be to measure atmospheric pressure of  $1 \text{ bar}$ ?

[&gt;10.19m]

12. What is the maximum gauge pressure of water that can be measured by a Piezometer of height 1.5m? And if the liquid had a relative density of 8.5 what would the maximum measurable gauge pressure?

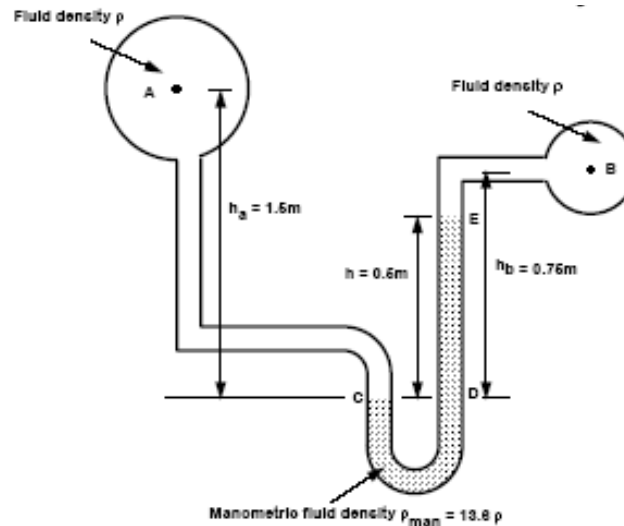
13. Using a u-tube manometer to measure gauge pressure of fluid density  $\rho = 700 \text{ kg/m}^3$ , and the manometric fluid is mercury, with a relative density of 13.6.

What is the gauge pressure if:

a)  $h_1 = 0.4\text{m}$  and  $h_2 = 0.9\text{m}$ ?

b)  $h_1$  stayed the same but  $h_2 = -0.1\text{m}$ ?

14. In the figure below two pipes containing the same fluid of density  $\rho = 990 \text{ kg/m}^3$  are connected using a u-tube manometer. What is the pressure between the two pipes if the manometer contains fluid of relative density 13.6?



15. An inclined manometer is required to measure an air pressure of 3mm of water to an accuracy of  $\pm 3\%$ . The inclined arm is 8mm in diameter and the larger arm has a diameter of 24mm. The manometric fluid has density  $740 \text{ kg/m}^3$  and the scale may be read to  $\pm 0.5\text{mm}$ . What is the angle required to ensure the desired accuracy may be achieved?

[7.6 ]